Case - Commodities
Due: 11.24.2014 (start of class #4)

Note:

Case reports should be done in groups (5 people). Case reports should be one page (including graphics) and show some careful and creative presentations. All of our cases are along the lines of the boss as asked about “X” and this is your chance to show your quantitative skills.

Commodities

You are working at Liquid Field Advisors. LFA is in the advice business offering advice to asset managers around the world. Lots of questions have been rolling in over the last few months about commodities. Is this an asset class that should be considered in building a portfolio. The partner in charge of the New York office has dumped this on your desk – “Commodities. What about them?” She is not one to be overly detailed with instructions. Fortunately, you have gathered some data and have a plan (see below). The high level question – To Do 1: [What are the key properties of commodities as an asset class?]

Data

The data consists of monthly futures prices (at various horizons; prices are shown as log. The current oil price is about $85 per bbl. So this is listed as 4.442651 = log(85.00)) and one-month holding period return for each of the futures contract (commodity and horizon) given. See the next page for a note on how we calculate returns here. To Do 2: [Pick ONE Commodity from:]

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1http://www.hedgefundnamegenerator.com/ and I seem to have noticed that you are working at a different firm each week – quite the resume-builder.
The OJ and Wheat data have a few more missing entries. I am not sure why and will investigate.

Feel free to use more than one or use them all.

**To Do 3:** [Calculate some descriptive statistics]. Describe the data. For example, look at the term structure mean (average price by horizon), volatility (average standard deviation by horizon). You can also look at how these change over sub-samples (pre and post 2004, for example). Take a look at the average slope of the futures curve (say the one year less the 3 month – depending on the commodity, other definitions will make more sense).

**To Do 4:** [Describe the returns data.] Do these earn a risk premium? Are they correlated to any of the risk factors we have been using?

**To Do 5:** [Are returns predictable in ways similar/different to equities?] This is a harder question. But try this to get yourself rolling. Run a regression:

\[ F(t + h, n - h) - F(t, 1) = a + b(F(t, n) - F(t, 1)) + error \]

The idea: Does the futures price at date \( t \) on a contract of horizon \( n \) predict the (spot \( h = 0 \)) price at date \( t + n \). If yes, what does that mean? And! If no, what does that mean? (Hint: “no” is good if you are an asset manager). You can try this regression for various prediction windows \( (n) \) and for different maturity contracts \( (h) \). (Also related, the data are state as log of the prices and, as you see in the note below, returns are closely related to the difference in log prices).
Quick reminder and refresher on derivatives and returns

We will cover some of this in class. But this is a reminder or introduction. It is not needed to do the assignment per se. But some details will help.

The main new item here will be the definition of returns. If you have not seen commodities before and want more information, any standard finance text book will bring you up to speed pretty quickly. Let me know if you need a pointer to a good reference. Secondly, specific institutional details in commodity markets can be very picky. For example, just what is a barrel of oil? Exchange traded contracts (and most over-the-counter contracts) will be very clear about the quality, quantity, and destination for delivery. Each market will have its own quirks. You can see lots of details at http://www.cmegroup.com/company/ nymex.html. In addition, the depth and liquidity (ease and cost of trade) varies across commodities and across maturities. As a general rule: (1) Very-short-horizon contracts can have odd behavior due to the nuances of delivery; (2) Shorter horizon contracts are more liquid.

Definitions:

Spot price, $P_t$, is the price paid today, $t$, for immediate delivery (e.g., buying gasoline at the gas station). For many commodities this is a defined price of a very specific commodity. The most commonly quoted spot price of oil is WTI. This means “West Texas Intermediate Oil Delivered to Cushing, Oklahoma.”

Forward Contract, $f_{t,n}$, is the forward price as of date $t$. It is the agreed to price for delivery at date $t + n$. All the data we have are monthly, so think of $t$ as month $t$ and $t + n$ is $n$ month horizon. In a forward contract, the date-$t$ value is zero. That is; no cash changes hands right now. At the terminal date, $t + n$, the payoff to the long side is $P_{t+n} - f_{t,n}$. If you are long you pay the $f_{t,n}$ and get yourself one unit of the commodity whose current price is $P_{t+n}$. Notice that unlike a call option, the payoff at date $t + n$ can be positive or negative.

Futures Contract is very similar to a forward contract. The data we are working with here and most exchange-traded commodity contracts are futures contracts. The idea of a futures contract is the same; we agree on a futures price, $F_{t,n}$, at date $t$ that will be “paid” in exchange for the spot good at date $t + n$. The key difference is that cash payments will be made or received at each interim date. One (maybe very big) payment at $t + n$ of $P_{t+n} - f_{t,n}$ can involve lots of
credit risk. Some of the oil contracts in our data last 5 or 10 years. So a futures contract breaks this big payment into many small pieces. In practice, the futures contracts are settled daily. However, we will treat them as settling each month. (This is an approximation. But it is unimportant given our interest in holding period returns).

To describe the cash flows: At date \( t \), enter into a long contract at price \( F_{t,n} \). Suppose \( t \) is now January and \( n = 5 \) has a delivery date of June. The date \( t \) cash-flow is zero. At date \( t + 1 \) the contract is settled (paid out) as \( F_{t+1,n-1} - F_{t,n} \). One month from now, your \( n \) horizon contract has a horizon of \( n - 1 \). It is now \( t + 1 \) February and June delivery is \( n - 1 = 4 \) months away. You pay (or receive) the cash on the change in contract-price. This is, of course, recursive. It has the boundary condition that \( F_{t+n,0} = P_{t+n} \).

**Pricing**

We can chat more about this in class. But pricing these contracts is a skill you are quite familiar with. It works exactly the same as with bonds. The key is that at date \( t \) the “value” of the contract is zero. We choose the futures price so that the two parties (the long side and the short side) both agree that the current value is zero. Let \( m_{t+1} \) be the (one-step ahead) pricing kernel or stochastic discount factor (recall out first lecture). For any arbitrary random cash flow, \( x_{t+1} \), the date \( t \) value is \( E_t[m_{t+1}x_{t+1}] \) (where the \( E_t \) is the expectation conditional on information we have at date \( t \)). Applying this to define \( F_{t,n} \), as the price agreed to in period \( t \) for delivery at period \( t + n \). Futures prices satisfy

\[
0 = E_t[m_{t+1}(F_{t+1,n-1} - F_{t,n})]
\]

This implies the “value” of the contract is always zero. It is zero at the initial date by construction and zero at all interim dates since you pay (or receive) cash as the futures prices fluctuate. To close this off, we need the boundary condition that \( F_{t,0} = P_t \).

If you want a simple example to poke at, set \( m_{t+1} = 0.95 \) (everyone is risk neutral and the interest rate is about 5%). In this very simple case, notice that \( F_{t,n} = E[P_{t+n}] \). We will soon see that this is not true in the data. Risk premiums are big and variable.

(You can convert that easily to expectations under the "Q" measure. Just recall that \( q_{t+1}(s)/p_{t+1}(s) = m_{t+1}(s)/B_t \). Where \( B_t = (1 + r_{f,t})^{-1} \approx 0.98 \).)
Returns

Are commodities a good “asset class”? This is a question about the characteristic of holding returns. Since a futures contract is a zero-wealth position, the standard is to define the return as the “fully collateralized holding return” as follows. The idea is that you enter into a long side of a futures contract, $F_{t,n}$. This involves no cash. So place $F_{t,n}$ in a bond (usually held by your broker). This is “fully collateralized” as the future payoff on the position in the future plus the bond is bounded at zero. Just like when you buy a share of equity, the payoff is always above zero. We are not using any leverage in this position.

The fully collateralized return (and we work in logs throughout, so the continuously compounded return is the natural definition) involves purchasing $F_{t,n}$ of a one-month bond (with interest rate $r^f_{t+1}$) and entering into the $t + n$ futures contract with agreed price $F_{t,n}$ at date $t$. Cash-flows at date $t + 1$ come from the risk-free rate, $F^h_t \exp(r^f_{t+1})$ and the change in futures prices $F_{t+1,h-1} - F_{t,h}$. So the continuously compounded return on this position is,

$$r^n_{t+1} = \log \left( \frac{F_{t+1,n-1} - F_{t,n} + (F_{t,n}(\exp r^f_{t+1}))}{F_{t,n}} \right)$$

We are interested in risk premiums, so will look at the rate in excess returns of the risk-free rate $r^f_{t+1}$. (Note the dating convention: that is the return earned from date $t$ to date $t + 1$. For the risk-free rate, this is a constant known at date $t$). Defining things this way means that excess returns are approximately equal to the log-change in futures prices.

$$r^n_{t+1} - r^f_{t+1} \approx \log F_{t+1,n-1} - \log F_{t,n}$$